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Hamiltonian BRST deformation of a class of n-dimensional BF-type theories

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Abstract: Consistent hamiltonian interactions that can be added to an abelian free BF-type class of theories in any $n \geq 4$ spacetime dimensions are constructed in the framework of the hamiltonian BRST deformation based on cohomological techniques. The resulting model is an interacting field theory in higher dimensions with an open algebra of on-shell reducible first-class constraints. We argue that the hamiltonian couplings are related to a natural structure of Poisson manifold on the target space.

Keywords: BRST Quantization, BRST Symmetry.

Contents

1. Introduction

The hamiltonian BRST symmetry [1, 2] has the advantage of a proper implementation in quantum mechanics [1, chapter 14], and also of an appropriate correlation with the canonical quantization methods [3]. The understanding of this symmetry on cohomological grounds made possible a unitary approach to many problems in gauge field theory, such as the hamiltonian analysis of anomalies [4], the precise relation between local lagrangian and hamiltonian BRST cohomologies [5], and, recently, the problem of obtaining consistent hamiltonian interactions in gauge theories by means of the deformation theory [6, 7, 8].

In this paper we investigate the consistent hamiltonian BRST deformations in any spacetime dimension $n \geq 4$ of a free abelian topological field theory[1] of BF-type [9] involving a set of scalar fields, two collections of one-forms and a system of two-forms. Actually, this work generalizes our previous results in two and four spacetime dimensions [7, 8]. We show that the resulting interactions are accurately described by a topological field theory with an open algebra of first-class constraints, that can be interpreted in terms of a Poisson structure present in various models of two-dimensional gravity [10]–[13]. The analysis of Poisson Sigma Models, including their relationship to two-dimensional gravity and the study of classical solutions, can be found in [14]–[19] (see also [20]). In view of this interpre-

[1]By 'topological field theory' we mean a gauge theory whose moduli space is finite dimensional, but nonzero, in contrast to 'topological invariant theories', whose moduli space is zero dimensional.

tation, we believe that the open problem of how to approach the hamiltonian quantization of gravity without string theory might benefit from the construction of consistently interacting hamiltonian gauge theories (in higher spacetime dimensions) with special classes of open algebras.

The plan of the paper is the following. Section 2 briefly reviews the problem of constructing consistent hamiltonian interactions in the framework of the BRST formalism, which reduces to solving two towers of equations that describe the deformation of the BRST charge, respectively, of the BRST-invariant hamiltonian associated with a given "free" first-class theory at various orders in the coupling constant. In section 3 we determine the hamiltonian BRST symmetry (s) of the free theory under study in $n \geq 4$ spacetime dimensions, which splits as the sum between the Koszul-Tate differential and the exterior derivative along the gauge orbits. This model is abelian and $(n-2)$-stage reducible, the reducibility relations holding off-shell. Next, we solve the main equations governing the hamiltonian deformation procedure on behalf of the BRST cohomology of the free theory. In section 4 we initially compute, using specific cohomological techniques, the first-order deformation of the BRST charge, which lies in the cohomological space of s modulo the spatial part of the exterior spacetime derivative ($\tilde{d}$) at ghost number one, $H^1(s|\tilde{d})$. The first-order deformation of the BRST charge stops at antighost number $(n-1)$ and contains two sorts of arbitrary functions involving only the undifferentiated scalar fields. Its consistency reveals one 'two-tensor' (in the collection indices) depending on the scalar fields, that must be antisymmetric and fulfill a certain identity. Under these conditions, all the other deformations, of order two and higher, can be taken to vanish, and thus the BRST charge of the interacting model that is consistent to all orders in the deformation parameter is fully output. Section 5 solves the problem of generating the deformed BRST-invariant hamiltonian, which can be taken nonzero only at the first order in the coupling constant. With the help of these deformed quantities, in section 6 we identify the interacting gauge theory, which displays an open algebra of constraints. The deformed first-class constraints are of course reducible, but the reducibility relations hold on-shell. In the meantime, we can interpret the 'two-tensor' mentioned in the above in terms of a target space parametrized by the scalar fields and endowed with a natural structure of Poisson manifold. Section 7 contains the main conclusions of the present paper.

2. Main equations of the hamiltonian deformation procedure

It has been shown in [6] that the problem of constructing consistent hamiltonian interactions in theories with first-class constraints can be equivalently reformulated as a deformation problem of the BRST charge Ω_0 and of the BRST-invariant hamiltonian $H_{0\mathrm{B}}$ of a given "free" first-class theory. More precisely, if the interactions can be consistently constructed, then the "free" BRST charge can be deformed into

$$\begin{aligned} \Omega_0 \rightarrow \Omega &= \Omega_0 + g \int d^{n-1}x\, \omega_1 + g^2 \int d^{n-1}x\, \omega_2 + O\left(g^3\right) \\ &= \Omega_0 + g\Omega_1 + g^2\Omega_2 + O\left(g^3\right), \end{aligned} \tag{2.1}$$

where the BRST charge of the interacting theory, Ω, must satisfy the equation

$$[\Omega, \Omega] = 0\,. \tag{2.2}$$

The symbol $[,]$ signifies the Poisson (Dirac) bracket. The last formula projected on various powers in the deformation parameter g leads to the tower of equations

$$[\Omega_0, \Omega_0] = 0\,, \tag{2.3}$$
$$2\,[\Omega_0, \Omega_1] = 0\,, \tag{2.4}$$
$$2\,[\Omega_0, \Omega_2] + [\Omega_1, \Omega_1] = 0\,, \tag{2.5}$$
$$\vdots$$

In a similar manner, we deform the BRST-invariant hamiltonian of the "free" theory

$$\begin{aligned} H_{0\mathrm{B}} \to H_{\mathrm{B}} &= H_{0\mathrm{B}} + g \int d^{n-1}x\, h_1 + g^2 \int d^{n-1}x\, h_2 + O\left(g^3\right) \\ &= H_{0\mathrm{B}} + gH_1 + g^2 H_2 + O\left(g^3\right), \end{aligned} \tag{2.6}$$

and require that it stands for the BRST-invariant hamiltonian of the coupled system

$$[H_{\mathrm{B}}, \Omega] = 0\,. \tag{2.7}$$

The decomposition of the relation (2.7) according to the various orders in the coupling constant reveals a new tower of equations

$$[H_{0\mathrm{B}}, \Omega_0] = 0\,, \tag{2.8}$$
$$[H_1, \Omega_0] + [H_{0\mathrm{B}}, \Omega_1] = 0\,, \tag{2.9}$$
$$[H_2, \Omega_0] + [H_1, \Omega_1] + [H_{0\mathrm{B}}, \Omega_2] = 0\,, \tag{2.10}$$
$$\vdots$$

While the equations (2.3) and (2.8) are satisfied since Ω_0 and $H_{0\mathrm{B}}$ are by hypothesis the BRST charge, respectively, the BRST-invariant hamiltonian of the "free" theory, the resolution of the remaining equations ((2.4)–(2.5), etc. and (2.9)–(2.10), etc.) by means of cohomological techniques provides the hamiltonian BRST description of the deformed gauge theory.

3. Free BRST symmetry

Our starting point is a free topological field theory of BF-type in any $n \geq 4$ spacetime dimension, that involves two types of one-forms, a collection of scalar fields, and a system of two-forms, described by the lagrangian action

$$S_0\left[A_\mu^a, H_\mu^a, \varphi_a, B_a^{\mu\nu}\right] = \int d^n x \left(H_\mu^a \partial^\mu \varphi_a + \frac{1}{2} B_a^{\mu\nu} \partial_{[\mu} A_{\nu]}^a\right), \tag{3.1}$$

where here and in the sequel the notation $[\mu\nu]$ signifies antisymmetry with respect to the indices between brackets. The above action is invariant under the gauge transformations

$$\delta_\epsilon A^a_\mu = \partial_\mu \epsilon^a, \ \delta_\epsilon H^a_\mu = \partial^\nu \epsilon^a_{\mu\nu}, \ \delta_\epsilon \varphi_a = 0, \ \delta_\epsilon B^{\mu\nu}_a = \partial_\rho \epsilon^{\mu\nu\rho}_a \,, \tag{3.2}$$

which are off-shell $(n-2)$-stage reducible, where the gauge parameters ϵ^a, $\epsilon^a_{\mu\nu}$ and $\epsilon^{\mu\nu\rho}_a$ are bosonic, the last two sets being completely antisymmetric.

We denote by $\left(\pi^\mu_a, \pi^a_{\mu\nu}, p^\mu_a, p^a\right)$ the canonical momenta respectively conjugated to the fields $\left(A^a_\mu, B^{\mu\nu}_a, H^a_\mu, \varphi_a\right)$. By performing the canonical analysis and by eliminating the second-class constraints (the coordinates of the reduced phase-space are $z^A = (\pi^0_a, A^a_\mu, B^{\mu\nu}_a, p^i_a, H^a_\mu, \pi^a_{ij}, \varphi_a))$, we are left with a system subject only to the first-class constraints

$$G^{(1)}_a \equiv \pi^0_a \approx 0\,, \qquad G^{(2)}_a \equiv -\partial_i B^{0i}_a \approx 0\,, \tag{3.3}$$

$$G^{(1)a}_{ij} \equiv 2\pi^a_{ij} \approx 0\,, \qquad G^{(2)a}_{ij} \equiv -\partial_{[i} A^a_{j]} \approx 0\,, \tag{3.4}$$

$$\gamma^{(1)i}_a \equiv -p^i_a \approx 0\,, \qquad \gamma^{(2)i}_a \equiv \partial^i \varphi_a \approx 0\,, \tag{3.5}$$

and displaying the first-class hamiltonian

$$H_0 = \int d^{n-1}x \left(-H^a_i \gamma^{(2)i}_a + \frac{1}{2} B^{ij}_a G^{(2)a}_{ij} + A^a_0 G^{(2)}_a\right), \tag{3.6}$$

in terms of the non-vanishing fundamental Dirac brackets

$$\left[\pi^0_a(t,\mathbf{x}), A^b_0(t,\mathbf{y})\right] = -\delta^b_a \delta^{n-1}\left(\mathbf{x}-\mathbf{y}\right), \tag{3.7}$$

$$\left[B^{0i}_a(t,\mathbf{x}), A^b_j(t,\mathbf{y})\right] = -\delta^i_j \delta^b_a \delta^{n-1}\left(\mathbf{x}-\mathbf{y}\right), \tag{3.8}$$

$$\left[H^a_0(t,\mathbf{x}), \varphi_b(t,\mathbf{y})\right] = -\delta^a_b \delta^{n-1}\left(\mathbf{x}-\mathbf{y}\right), \tag{3.9}$$

$$\left[\pi^a_{ij}(t,\mathbf{x}), B^{kl}_b(t,\mathbf{y})\right] = -\frac{1}{2}\delta^{[k}_i \delta^{l]}_j \delta^a_b \delta^{n-1}\left(\mathbf{x}-\mathbf{y}\right), \tag{3.10}$$

$$\left[p^i_a(t,\mathbf{x}), H^b_j(t,\mathbf{y})\right] = -\delta^i_j \delta^b_a \delta^{n-1}\left(\mathbf{x}-\mathbf{y}\right). \tag{3.11}$$

The above constraints are abelian, while the remaining gauge algebra relations are expressed by

$$\left[H_0, G^{(1)}_a\right] = G^{(2)}_a\,, \qquad \left[H_0, G^{(2)}_a\right] = 0\,, \tag{3.12}$$

$$\left[H_0, G^{(1)a}_{ij}\right] = G^{(2)a}_{ij}\,, \qquad \left[H_0, G^{(2)a}_{ij}\right] = 0\,, \tag{3.13}$$

$$\left[H_0, \gamma^{(1)i}_a\right] = \gamma^{(2)i}_a\,, \qquad \left[H_0, \gamma^{(2)i}_a\right] = 0\,. \tag{3.14}$$

The constraint functions $G^{(2)a}_{ij}$ are off-shell $(n-3)$-stage reducible, with the reducibility functions (of order $(k-2)$) given by

$$\left(Z^a_{i_1 i_2 \cdots i_k}\right)^{j_1 \cdots j_{k-1}}_b = \frac{(-)^{k-1}}{(k-1)!}\delta^a_b \partial_{[i_1} \delta^{j_1}_{i_2} \cdots \delta^{j_{k-1}}_{i_k]}\,, \qquad k = 3, \ldots, n-1\,, \tag{3.15}$$

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while the constraint functions $\gamma_a^{(2)i}$ are off-shell $(n-2)$-stage reducible, the associated reducibility functions (of order $(k-1)$) being

$$\left(Z_a^{i_1 i_2 \cdots i_k}\right)^b_{j_1 \cdots j_{k-1}} = \frac{(-)^{k-1}}{(k-1)!}\delta_a^b \partial^{[i_1}\delta^{i_2}_{j_1}\cdots\delta^{i_k]}_{j_{k-1}}\,, \qquad k=2,\ldots,n-1\,. \tag{3.16}$$

The hamiltonian BRST formalism requires the introduction of the ghosts

$$\eta^{a_0} = \left(\eta^{(1)a}, \eta^a, \eta_a^{(1)ij}, \eta_a^{ij}, C_i^{(1)a}, C_i^a\right), \tag{3.17}$$

$$\eta^{a_k} = \left(C^a_{i_1\cdots i_{k+1}}, \eta_a^{i_1\cdots i_{k+2}}\right), \qquad k=1,\ldots,n-3\,, \tag{3.18}$$

$$\eta^{a_{n-2}} = \left(C^a_{i_1\cdots i_{n-1}}\right), \tag{3.19}$$

together with their conjugated antighosts

$$\mathcal{P}_{a_0} = \left(\mathcal{P}_a^{(1)}, \mathcal{P}_a, \mathcal{P}_{ij}^{(1)a}, \mathcal{P}_{ij}^a, P_a^{(1)i}, P_a^i\right), \tag{3.20}$$

$$\mathcal{P}_{a_k} = \left(P_a^{i_1\cdots i_{k+1}}, \mathcal{P}^a_{i_1\cdots i_{k+2}}\right), \qquad k=1,\ldots,n-3\,, \tag{3.21}$$

$$\mathcal{P}_{a_{n-2}} = \left(P_a^{i_1 i_2\cdots i_{n-1}}\right). \tag{3.22}$$

The first set of ghosts respectively corresponds to the first-class constraints (3.3)–(3.5), while the other two are due to the reducibility of the first-class constraint functions. The fields η^{a_0} in (3.17) are fermionic and of ghost number one, the fields η^{a_k} in (3.18) possess ghost number $(k+1)$ and Grassmann parity $(k+1) \bmod 2$, while those in (3.19) have ghost number $(n-1)$ and Grassmann parity $(n-1) \bmod 2$. The ghost number and Grassmann parity of the antighosts follow from the general rules of the standard hamiltonian BRST formalism. The ghost number is defined in usual manner as the difference between the pure ghost number (pgh) and the antighost number (antigh), where

$$\mathrm{pgh}\left(z^A\right)=0\,, \qquad \mathrm{pgh}\left(\eta^{a_0}\right)=1\,, \qquad \mathrm{pgh}\left(\mathcal{P}_{a_0}\right)=0\,, \tag{3.23}$$

$$\mathrm{pgh}\left(\eta^{a_k}\right)=k+1\,, \qquad \mathrm{pgh}\left(\mathcal{P}_{a_k}\right)=0\,, \qquad k=1,\ldots,n-3\,, \tag{3.24}$$

$$\mathrm{pgh}\left(\eta^{a_{n-2}}\right)=n-1\,, \qquad \mathrm{pgh}\left(\mathcal{P}_{a_{n-2}}\right)=0\,, \tag{3.25}$$

$$\mathrm{antigh}\left(z^A\right)=0\,, \qquad \mathrm{antigh}\left(\eta^{a_0}\right)=0\,, \qquad \mathrm{antigh}\left(\mathcal{P}_{a_0}\right)=1\,, \tag{3.26}$$

$$\mathrm{antigh}\left(\eta^{a_k}\right)=0\,, \qquad \mathrm{antigh}\left(\mathcal{P}_{a_k}\right)=k+1\,, \qquad k=1,\ldots,n-3\,, \tag{3.27}$$

$$\mathrm{antigh}\left(\eta^{a_{n-2}}\right)=0\,, \qquad \mathrm{antigh}\left(\mathcal{P}_{a_{n-2}}\right)=n-1\,. \tag{3.28}$$

The BRST charge of this free model takes the form

$$\begin{aligned}\Omega_0 = \int d^{n-1}x\Bigg(&\eta^{(1)a}G_a^{(1)} + \eta^a G_a^{(2)} + \eta_a^{(1)ij}G_{ij}^{(1)a} + \eta_a^{ij}G_{ij}^{(2)a} + C_i^{(1)a}\gamma_a^{(1)i} + C_i^a\gamma_a^{(2)i}\\ &+\sum_{k=3}^{n-1}(-)^{k-1}\,\eta_a^{i_1 i_2\cdots i_k}\partial_{[i_1}\mathcal{P}^a_{i_2\cdots i_k]} + \sum_{k=2}^{n-1}(-)^{k-1}\,C^a_{i_1 i_2\cdots i_k}\partial^{[i_1}P_a^{i_2\cdots i_k]}\Bigg),\end{aligned} \tag{3.29}$$

while the corresponding BRST-invariant hamiltonian, which is solution to the equation (2.8), is expressed like

$$H_{0\mathrm{B}} = H_0 + \int d^{n-1}x\left(\eta^{(1)a}\mathcal{P}_a + \eta_a^{(1)ij}\mathcal{P}_{ij}^a + C_i^{(1)a}P_a^i\right). \tag{3.30}$$

In general, any function F with $\mathrm{gh}\,(F) = 0$ that fulfills $[F, \Omega_0] = 0$ is called BRST observable. Since the investigated theory has no physical degrees of freedom, its BRST observables are BRST-exact, $F = [M_0, \Omega_0]$, for some M_0 with $\mathrm{gh}\,(M_0) = -1$. In consequence, the BRST-invariant hamiltonian will also be BRST-exact

$$H_{0\mathrm{B}} = [K_0, \Omega_0]\,, \tag{3.31}$$

where

$$K_0 = \int d^{n-1}x \left(H^a_i P^i_a - \frac{1}{2} B^{ij}_a \mathcal{P}^a_{ij} - A^a_0 \mathcal{P}_a \right). \tag{3.32}$$

The BRST charge encodes all the information on the gauge structure of the first-class constraints. We remark that in our case the free BRST charge (3.29) breaks into terms with antighost numbers ranging from zero to $(n-2)$. The pieces with antighost number zero contain the first-class constraint functions (3.3)–(3.5). If the algebra of the first-class constraints is non-abelian, then there appear terms linear in the antighost number one antighosts and quadratic in the pure ghost number one ghosts. The absence of such terms in our case reflects that the first-class constraints are abelian. The elements from (3.29) with higher antighost number give us information on the reducibility functions (3.15)–(3.16). If the reducibility relations held on-shell, then there would appear components linear in the ghosts for ghosts (ghosts of pure ghost number strictly greater than one) and at least quadratic in the various antighosts. Such pieces are not present in (3.29) since the reducibility relations hold off-shell. Other possible components in the BRST charge offer information on the higher-order structure functions related to the first-class constraints. There are no such terms in (3.29), as a consequence of the fact that all higher-order structure functions vanish for the free theory analysed in the above. On the other hand, the BRST-invariant hamiltonian (3.30) decomposes into pieces of antighost number zero and one. The element of antighost number zero is nothing but the first-class hamiltonian. The terms of antighost number one underlies the brackets between the first-class hamiltonian and the first-class constraints (the relations (3.12)–(3.14)). For a generic theory, there might appear pieces of higher antighost number as well, that provide information on the higher-order structure functions related to the first-class hamiltonian. By deforming the BRST charge and the BRST-invariant hamiltonian, one deforms everything, namely, the first-class constraints, their algebra, the reducibility relations and their behaviour, the first-class hamiltonian, its brackets with the new first-class constraints, etc.

The BRST symmetry of the free theory, $s\cdot = [\cdot, \Omega_0]$, splits as

$$s = \delta + \gamma\,, \tag{3.33}$$

where δ denotes the Koszul-Tate differential ($\mathrm{antigh}\,(\delta) = -1$, $\mathrm{pgh}\,(\delta) = 0$), and γ represents the exterior longitudinal derivative ($\mathrm{antigh}\,(\gamma) = 0$, $\mathrm{pgh}\,(\gamma) = 1$). These two operators act on the variables from the BRST complex like

$$\delta z^A = 0\,, \qquad \delta \eta^{a_k} = 0\,, \qquad k = 0, \ldots, n-2\,, \tag{3.34}$$

$$\delta \mathcal{P}^{(1)}_a = -\pi^0_a\,, \qquad \delta \mathcal{P}_a = \partial_i B^{0i}_a\,, \qquad \delta P^{(1)i}_a = p^i_a\,, \qquad \delta P^i_a = -\partial^i \varphi_a, \tag{3.35}$$

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$$\delta\mathcal{P}^{(1)a}_{ij} = -2\pi^a_{ij}\,, \qquad \delta\mathcal{P}^a_{ij} = \partial_{[i}A^a_{j]}, \tag{3.36}$$

$$\delta P_a^{i_1 i_2\cdots i_k} = (-)^k\,\partial^{[i_1}P_a^{i_2\cdots i_k]}\,, \qquad k = 2,\ldots,n-1, \tag{3.37}$$

$$\delta\mathcal{P}^a_{i_1 i_2\cdots i_k} = (-)^k\,\partial_{[i_1}\mathcal{P}^a_{i_2\cdots i_k]}\,, \qquad k = 3,\ldots,n-1, \tag{3.38}$$

$$\gamma A^a_i = \partial_i\eta^a\,, \quad \gamma A^a_0 = \eta^{(1)a}\,, \quad \gamma\varphi_a = 0\,, \quad \gamma\pi^0_a = 0\,, \quad \gamma p^i_a = 0\,, \quad \gamma\pi^a_{ij} = 0\,, \tag{3.39}$$

$$\gamma B^{0i}_a = 2\partial_j\eta^{ij}_a\,, \qquad \gamma B^{ij}_a = 2\eta^{(1)ij}_a\,, \qquad \gamma H^a_i = -C^{(1)a}_i\,, \qquad \gamma H^a_0 = \partial^i C^a_i\,, \tag{3.40}$$

$$\gamma\eta^{(1)a} = \gamma\eta^a = \gamma C^{(1)a}_i = \gamma\eta^{(1)ij}_a = 0\,, \tag{3.41}$$

$$\gamma\eta^{ij}_a = 3\partial_k\eta^{ijk}_a\,, \qquad \gamma C^a_i = 2\partial^j C^a_{ij}\,, \tag{3.42}$$

$$\gamma\eta^{i_1\cdots i_k}_a = (k+1)\,\partial_i\eta^{ii_1\cdots i_k}_a\,, \qquad k = 3,\ldots,n-2\,, \tag{3.43}$$

$$\gamma C^a_{i_1\cdots i_k} = -\,(k+1)\,\partial^i C^a_{ii_1\cdots i_k}\,, \qquad k = 2,\ldots,n-2\,, \tag{3.44}$$

$$\gamma\eta^{i_1\cdots i_{n-1}}_a = 0\,, \qquad \gamma C^a_{i_1\cdots i_{n-1}} = 0\,, \tag{3.45}$$

$$\gamma\mathcal{P}_{a_k} = 0\,, \qquad k = 0,\ldots,n-2\,. \tag{3.46}$$

The last formulas will be employed in the next section at the deformation of the free theory.

4. Deformation of the BRST charge

In this section we solve the equations (2.4)–(2.5), etc., that govern the deformation of the BRST charge in the case of the free model under study by relying on cohomological techniques. We will focus only on both local and spacetime-dimension independent deformations. As a result, we find that only the first-order deformation is non-trivial, while its consistency is equivalent to the existence of a 'two-tensor' (in the collection indices) depending on the undifferentiated scalar fields, that must be antisymmetric and fulfill a certain identity.

4.1 First-order deformation

Initially, we solve the equation (2.4), which is responsible for the first-order deformation of the BRST charge. Using the notations from (2.1), it takes the local form

$$s\omega_1 = \partial_i j^i\,, \tag{4.1}$$

for some local j^i. In order to investigate this equation, we develop ω_1 according to the antighost number and suppose that the development stops at a finite order

$$\omega_1 = \overset{(0)}{\omega}_1 + \overset{(1)}{\omega}_1 + \cdots + \overset{(J)}{\omega}_1\,, \qquad \text{antigh}\left(\overset{(I)}{\omega}_1\right) = I\,, \qquad \text{gh}\left(\overset{(I)}{\omega}_1\right) = 1\,, \tag{4.2}$$

where the last term can be assumed to be annihilated by γ,

$$\gamma\,\overset{(J)}{\omega}_1 = 0\,. \tag{4.3}$$

Thus, we need to know the cohomology of γ, $H(\gamma)$, in order to determine the piece of highest antighost number in (4.2). From the actions (3.39)–(3.46) of γ acting on the BRST generators of the BRST complex, we remark that $H(\gamma)$ is generated by

$$\Phi^\alpha = \left(F^a_{ij} = \partial_{[i}A^a_{j]}, \varphi_a, \pi^0_a, p^i_a, \pi^a_{ij}, \partial_i B^{0i}_a\right), \tag{4.4}$$

(together with their spatial derivatives up to a finite order), by the antighosts (3.20)–(3.22) and their spatial derivatives up to a finite order, as well as by the undifferentiated ghosts η^a, $\eta_a^{i_1\cdots i_{n-1}}$ and $C^a_{i_1\cdots i_{n-1}}$. (The ghosts $\eta^{(1)a}$, $C_i^{(1)a}$ and $\eta_a^{(1)ij}$, although γ -invariant, are also γ-exact, and hence trivial in $H(\gamma)$. Same with respect to the spatial part of the spacetime derivatives of η^a, $\eta_a^{i_1\cdots i_{n-1}}$ and $C^a_{i_1\cdots i_{n-1}}$.) In this way, the general solution to the equation (4.3) can be written (up to a trivial term) as

$$\overset{(J)}{\omega}_1 = a_J\left([\Phi^\alpha],[\mathcal{P}_{a_k}]_{k=0,\ldots,n-2}\right) e^{J+1}\left(\eta^a,\eta_a^{i_1\cdots i_{n-1}},C^a_{i_1\cdots i_{n-1}}\right), \tag{4.5}$$

where $e^{J+1}(\eta^a,\eta_a^{i_1\cdots i_{n-1}},C^a_{i_1\cdots i_{n-1}})$ stand for the elements with pure ghost number equal to $(J+1)$ of a basis in the space of the polynomials in the corresponding ghosts, and obviously $\text{antigh}\,(a_J) = J$. The notation $f([q])$ signifies that f depends on q and its spatial derivatives up to a finite order.

The equation (4.1) projected on antighost number $(J-1)$ becomes

$$\delta \overset{(J)}{\omega}_1 + \gamma \overset{(J-1)}{\omega}_1 = \partial^i \overset{(J)}{m}_i, \tag{4.6}$$

and it shows that a necessary condition for the existence of $\overset{(J-1)}{\omega}_1$ is that the functions a_J from (4.5) belong to $H_J(\delta|\tilde{d})$, where the last notation means the homological space of the Koszul-Tate differential modulo the spatial part of the exterior spacetime derivative at antighost number J. Equivalently, these functions should satisfy the equation

$$\delta a_J = \partial^i n_i, \qquad \text{antigh}\,(n_i) = J-1. \tag{4.7}$$

Translating the lagrangian results from [21] at the hamiltonian level, as our model is $(n-2)$-order reducible and the constraint functions are linear in the reduced phase-space variables, we have that

$$H_K\left(\delta|\tilde{d}\right) = 0, \qquad \text{for } K > n-1. \tag{4.8}$$

Consequently, we can assume that $J = n-1$, and thus the development (4.2) stops after the first n terms

$$\omega_1 = \overset{(0)}{\omega}_1 + \overset{(1)}{\omega}_1 + \cdots + \overset{(n-1)}{\omega}_1, \tag{4.9}$$

with $\overset{(n-1)}{\omega}_1$ given by (4.5) with $J = n-1$ and a_{n-1} from $H_{n-1}(\delta|\tilde{d})$. After some computation, we find that the most general representative of $H_{n-1}(\delta|\tilde{d})$ is expressed like

$$\begin{aligned} a_{n-1}^{i_1\cdots i_{n-1}} &= \frac{\delta U}{\delta\varphi_a} P_a^{i_1\cdots i_{n-1}} + \sum_{p=2}^{n-1} \sum_{1\leq j_1\leq j_2\leq\cdots\leq j_p<n-1} \frac{\delta^p U}{\delta\varphi_{a_1}\delta\varphi_{a_2}\cdots\delta\varphi_{a_p}} \times \\ &\quad \times P_{a_1}^{[i_1\cdots i_{j_1}} P_{a_2}^{i_{j_1+1}\cdots i_{j_1+j_2}} \cdots P_{a_{p-1}}^{i_{j_1+\cdots+j_{p-2}+1}\cdots i_{j_1+\cdots+j_{p-1}}} P_{a_p}^{i_{j_1+\cdots+j_{p-1}+1}\cdots i_{n-1}]}, \end{aligned} \tag{4.10}$$

where U is an arbitrary function involving the undifferentiated scalar fields φ_a and

$$j_p = n-1-(j_1+j_2+\cdots+j_{p-1}). \tag{4.11}$$

Now, we can completely determine the last component in (4.9). The elements of

$$e^n\left(\eta^a, \eta_a^{i_1\cdots i_{n-1}}, C^a_{i_1\cdots i_{n-1}}\right)$$

can be written in the form

$$\eta^a C^b_{i_1\cdots i_{n-1}}, \eta^a\eta^b\eta_c^{i_1\cdots i_{n-1}}, \eta^{a_1}\eta^{a_2}\cdots\eta^{a_n}\,, \tag{4.12}$$

for $n \geq 4$.[2] It means that the piece of highest antighost number in the first-order deformation is determined once we 'glue' (4.10) to (4.12) like in (4.5). The last component in (4.12) needs the adjustment of a completely antisymmetric constant $K_{i_1\cdots i_{n-1}}$ in order to match (4.10), which can only be by 'covariance' arguments proportional to the spatial part of the completely antisymmetric symbol in n dimensions, $\varepsilon_{0i_1\cdots i_{n-1}}$. As mentioned in the above, we ask that the resulting deformations are independent of the spacetime dimension. If we add to the last component $\overset{(n-1)}{\omega}{}_1$ a term involving the completely antisymmetric symbol, this element will generate in the deformed BRST-invariant hamiltonian (by means of the equations (2.9)–(2.10), etc.) some pieces that contain this symbol, such that the lagrangian action of the interacting model will accordingly exhibit some vertices that break the PT-invariance. By imposing the PT-invariance at the level of the coupled gauge theory, the third element in (4.12) should be removed. Then, we finally obtain that

$$\overset{(n-1)}{\omega}{}_1 = -W_{ab}^{i_1\cdots i_{n-1}}\eta^a C^b_{i_1\cdots i_{n-1}} - \frac{(-)^n}{2}\left(M^c_{ab}\right)^{i_1\cdots i_{n-1}}\eta^a\eta^b\eta_{ci_1\cdots i_{n-1}}, \tag{4.13}$$

where $W_{ab}^{i_1\cdots i_{n-1}}$ and $\left(M^c_{ab}\right)^{i_1\cdots i_{n-1}}$ are obtained from $a_{n-1}^{i_1\cdots i_{n-1}}$ with the function U on the scalar fields replaced by a 'two-tensor' W_{ab}, respectively, a 'two–one-tensor' M^c_{ab}

$$\begin{aligned} W_{ab}^{i_1\cdots i_{n-1}} &= \frac{\delta W_{ab}}{\delta\varphi_a}P_a^{i_1\cdots i_{n-1}} + \sum_{p=2}^{n-1}\sum_{1\leq j_1\leq j_2\leq\cdots\leq j_p<n-1}\frac{\delta^p W_{ab}}{\delta\varphi_{a_1}\delta\varphi_{a_2}\cdots\delta\varphi_{a_p}}\times \qquad (4.14)\\ &\quad\times P_{a_1}^{[i_1\cdots i_{j_1}}P_{a_2}^{i_{j_1+1}\cdots i_{j_1+j_2}}\cdots P_{a_{p-1}}^{i_{j_1+\cdots+j_{p-2}+1}\cdots i_{j_1+\cdots+j_{p-1}}}P_{a_p}^{i_{j_1+\cdots+j_{p-1}+1}\cdots i_{n-1}]}\,,\\ \left(M^c_{ab}\right)^{i_1\cdots i_{n-1}} &= \frac{\delta M^c_{ab}}{\delta\varphi_a}P_a^{i_1\cdots i_{n-1}} + \sum_{p=2}^{n-1}\sum_{1\leq j_1\leq j_2\leq\cdots\leq j_p<n-1}\frac{\delta^p M^c_{ab}}{\delta\varphi_{a_1}\delta\varphi_{a_2}\cdots\delta\varphi_{a_p}}\times \qquad (4.15)\\ &\quad\times P_{a_1}^{[i_1\cdots i_{j_1}}P_{a_2}^{i_{j_1+1}\cdots i_{j_1+j_2}}\cdots P_{a_{p-1}}^{i_{j_1+\cdots+j_{p-2}+1}\cdots i_{j_1+\cdots+j_{p-1}}}P_{a_p}^{i_{j_1+\cdots+j_{p-1}+1}\cdots i_{n-1}]}\,. \end{aligned}$$

The notion of 'tensor' has no other significance for the moment than to emphasise that these functions in the dynamical fields φ_a carry more than one collection index differently positioned. Moreover, M^c_{ab} is antisymmetric in its lower indices due to the anticommutation of the fermionic ghosts η^a. The additional constants in (4.13) were introduced for convenience.

[2] For $n = 4$ there is an extra possibility because $\eta_a^{i_1\cdots i_{n-1}} \to \eta_a^{ijk}$, with $pgh\left(\eta_a^{ijk}\right) = 2$, and so we have a supplementary element of the basis in the ghosts at pure ghost number $n = 4$, namely, $\eta_a^{ijk}\eta_b^{i'j'k'}$. However, this element can be discarded [8], so finally (4.12) still covers all the investigated situations.

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Taking into account the actions (3.34)–(3.38) of the Koszul-Tate differential, we can prove the recursive relations

$$\delta a_k^{i_1 i_2 \cdots i_k} = (-)^k \, \partial^{[i_1} a_{k-1}^{i_2 \cdots i_k]}, \qquad k = 1, \ldots, n-1, \tag{4.16}$$

where for $k = 2, \ldots, n-2$ we have

$$\begin{aligned} a_k^{i_1 \cdots i_k} = & \frac{\delta U}{\delta \varphi_a} P_a^{i_1 \cdots i_k} + \sum_{q=2}^{k} \sum_{1 \leq j_1 \leq j_2 \leq \cdots \leq j_q < k} \frac{\delta^q U}{\delta \varphi_{a_1} \delta \varphi_{a_2} \cdots \delta \varphi_{a_q}} \times \\ & \times P_{a_1}^{[i_1 \cdots i_{j_1}} P_{a_2}^{i_{j_1+1} \cdots i_{j_1+j_2}} \cdots P_{a_{q-1}}^{i_{j_1+\cdots+j_{q-2}+1} \cdots i_{j_1+\cdots+j_{q-1}}} P_{a_q}^{i_{j_1+\cdots+j_{q-1}+1} \cdots i_k]}, \end{aligned} \tag{4.17}$$

and

$$a_1^i = \frac{\delta U}{\delta \varphi_a} P_a^i, \qquad a_0 = U. \tag{4.18}$$

In (4.17) we used the notation $j_q = k - (j_1 + j_2 + \cdots + j_{q-1})$. The formulas (4.16)–(4.18) hold in general, for any starting $a_{n-1}^{i_1 \cdots i_{n-1}}$ of the form (4.10), such that they are also valid for U replaced with W_{ab}, respectively, M_{ab}^c, like in (4.14)–(4.15). In the sequel, we will constantly use the above relations in order to infer the components of lower antighost number in Ω_1. Thus, the piece of antighost number $(n-2)$ from the first-order deformation, which is solution to the equation (4.6) with $J = n-1$, reads as

$$\begin{aligned} \stackrel{(n-2)}{\omega}_1 = & -W_{ab}^{i_1 \cdots i_{n-2}} \eta^a C_{i_1 \cdots i_{n-2}}^b + \frac{(-)^n}{2} \left(M_{ab}^c\right)^{i_1 \cdots i_{n-2}} \eta^a \eta^b \eta_{c i_1 \cdots i_{n-2}} - \\ & - C_{n-1}^1 W_{ab}^{i_1 \cdots i_{n-2}} A^{a i_{n-1}} C_{i_1 \cdots i_{n-1}}^b - \\ & - \sum_{k=3}^{n} (-)^k \, C_{n-1}^{k-1} W_{ab}^{i_1 \cdots i_{n-k}} \mathcal{P}^{a i_{n-k+1} \cdots i_{n-1}} C_{i_1 \cdots i_{n-1}}^b - \\ & - (-)^n \, C_{n-1}^1 \left(M_{ab}^c\right)^{i_1 \cdots i_{n-2}} A^{a i_{n-1}} \eta^b \eta_{c i_1 \cdots i_{n-1}} - \\ & - (-)^n \sum_{k=3}^{n} (-)^k \, C_{n-1}^{k-1} \left(M_{ab}^c\right)^{i_1 \cdots i_{n-k}} \mathcal{P}^{a i_{n-k+1} \cdots i_{n-1}} \eta^b \eta_{c i_1 \cdots i_{n-1}}. \end{aligned} \tag{4.19}$$

The equation (4.1) projected on antighost number $(n-3)$ becomes precisely the equation (4.6) for $J = n-2$, which further yields

$$\begin{aligned} \stackrel{(n-3)}{\omega}_1 = & -W_{ab}^{i_1 \cdots i_{n-3}} \eta^a C_{i_1 \cdots i_{n-3}}^b - \\ & - \frac{(-)^n}{2} \left(M_{ab}^c\right)^{i_1 \cdots i_{n-3}} \eta^a \eta^b \eta_{c i_1 \cdots i_{n-3}} - \\ & - C_{n-2}^1 W_{ab}^{i_1 \cdots i_{n-3}} A^{a i_{n-2}} C_{i_1 \cdots i_{n-2}}^b + \\ & + \sum_{k=4}^{n} (-)^k \, C_{n-2}^{k-2} W_{ab}^{i_1 \cdots i_{n-k}} \mathcal{P}^{a i_{n-k+1} \cdots i_{n-2}} C_{i_1 \cdots i_{n-2}}^b + \\ & + (-)^n \, C_{n-2}^1 \left(M_{ab}^c\right)^{i_1 \cdots i_{n-3}} A^{a i_{n-2}} \eta^b \eta_{c i_1 \cdots i_{n-2}} - \end{aligned}$$

$$
\begin{aligned}
&- (-)^n \sum_{k=4}^{n} (-)^k \, C_{n-2}^{k-2} \, (M_{ab}^c)^{i_1 \cdots i_{n-k}} \, \mathcal{P}^{a i_{n-k+1} \cdots i_{n-2}} \eta^b \eta_{c i_1 \cdots i_{n-2}} - \\
&- (-)^n \sum_{p=2}^{\left[\frac{n-2}{2}\right]} \sum_{q=p+1}^{n-p-1} (-)^q \, C_{n-1}^p C_{n-p-1}^q \, (M_{ab}^c)^{i_1 \cdots i_{n-p-q-1}} \times \\
&\times \mathcal{P}^{a j_1 \cdots j_q} \mathcal{P}^{b l_1 \cdots l_p} \eta_{c i_1 \cdots i_{n-p-q-1} j_1 \cdots j_q l_1 \cdots l_p} - \\
&- \frac{(-)^n}{2} \sum_{k=2}^{\left[\frac{n-1}{2}\right]} (-)^k \, C_{n-1}^k C_{n-k-1}^k \, (M_{ab}^c)^{i_1 \cdots i_{n-2k-1}} \times \\
&\times \mathcal{P}^{a j_1 \cdots j_k} \mathcal{P}^{b l_1 \cdots l_k} \eta_{c i_1 \cdots i_{n-2k-1} j_1 \cdots j_k l_1 \cdots l_k} + \\
&+ (-)^n \, C_{n-1}^2 \, (M_{ab}^c)^{i_1 \cdots i_{n-3}} \, A^{a i_{n-2}} A^{b i_{n-1}} \eta_{c i_1 i_2 \cdots i_{n-1}} \, .
\end{aligned} \tag{4.20}
$$

In the same manner, we can solve the equation (4.1) for any antighost number $(n-m)$, where $m = 4, \ldots, n-2$, and derive the solutions

$$
\begin{aligned}
\overset{(n-m)}{\omega}{}_1 = & -W_{ab}^{i_1 \cdots i_{n-m}} \eta^a C^b_{i_1 \cdots i_{n-m}} - \\
& - C_{n-m+1}^1 W_{ab}^{i_1 \cdots i_{n-m}} A^{a i_{n-m+1}} C^b_{i_1 \cdots i_{n-m+1}} - \\
& - \sum_{k=m+1}^{n} (-)^{k+m} \, C_{n-m+1}^{k-m+1} W_{ab}^{i_1 \cdots i_{n-k}} \mathcal{P}^{a i_{n-k+1} \cdots i_{n-m+1}} C^b_{i_1 \cdots i_{n-m+1}} + \\
& + \frac{(-)^{m+n}}{2} \, (M_{ab}^c)^{i_1 \cdots i_{n-m}} \, \eta^a \eta^b \eta_{c i_1 \cdots i_{n-m}} - \\
& - (-)^{m+n} \, C_{n-m+1}^1 \, (M_{ab}^c)^{i_1 \cdots i_{n-m}} \, A^{a i_{n-m+1}} \eta^b \eta_{c i_1 \cdots i_{n-m+1}} - \\
& - \sum_{k=m+1}^{n} (-)^{k+n} \, C_{n-m+1}^{k-m+1} \, (M_{ab}^c)^{i_1 \cdots i_{n-k}} \, \mathcal{P}^{a i_{n-k+1} \cdots i_{n-m+1}} \eta^b \eta_{c i_1 \cdots i_{n-m+1}} - \\
& - (-)^{m+n} \, C_{n-m+2}^2 \, (M_{ab}^c)^{i_1 \cdots i_{n-m}} \, A^{a i_{n-m+1}} A^{b i_{n-m+2}} \eta_{c i_1 \cdots i_{n-m+2}} \\
& + \frac{1}{2} \sum_{k=2}^{\left[\frac{n-m+2}{2}\right]} (-)^{k+n+m} \, C_{n-m+2}^k C_{n-m-k+2}^k \, (M_{ab}^c)^{i_1 \cdots i_{n-2k-m+2}} \times \\
& \times \mathcal{P}^{a j_1 \cdots j_k} \mathcal{P}^{b l_1 \cdots l_k} \eta_{c i_1 \cdots i_{n-2k-m+2} j_1 \cdots j_k l_1 \cdots l_k} + \\
& + \sum_{p=2}^{\left[\frac{n-m+1}{2}\right]} \sum_{q=p+1}^{n-m-p+2} (-)^{q+n+m} \, C_{n-m+2}^p C_{n-m-p+2}^q \, (M_{ab}^c)^{i_1 \cdots i_{n-m-p-q+2}} \times \\
& \times \mathcal{P}^{a j_1 \cdots j_q} \mathcal{P}^{b l_1 \cdots l_p} \eta_{c i_1 \cdots i_{n-m-p-q+2} j_1 \cdots j_q l_1 \cdots l_p} - \\
& - \sum_{k=m+1}^{n} (-)^{k+n} \, C_{n-m+1}^{k-m+1} C_{n-m+2}^1 \, (M_{ab}^c)^{i_1 \cdots i_{n-k}} \times \\
& \times \mathcal{P}^{a i_{n-k+1} \cdots i_{n-m+1}} A^{b i_{n-m+2}} \eta_{c i_1 i_2 \cdots i_{n-m+2}} \, .
\end{aligned} \tag{4.21}
$$

The equation (4.1) projected on antighost number one becomes exactly the equation (4.6) for $J=2$, which leads to

$$\begin{aligned}\overset{(1)}{\omega}_1 =& -\frac{\delta W_{ab}}{\delta\varphi_c}P^i_c\left(\eta^a C^b_i + 2A^{aj}C^b_{ij}\right) + W_{ab}\mathcal{P}^{aij}C^b_{ij} + \\ &+\frac{\delta M^c_{ab}}{\delta\varphi_d}P_{di}\left(\frac{1}{2}\eta^a\eta^b B^{0i}_c + 2A^a_j\eta^b\eta^{ij}_c + 3A^a_j A^b_k\eta^{ijk}_c\right) - \\ &- M^c_{ab}\left(\mathcal{P}^a_{ij}\eta^b\eta^{ij}_c + \mathcal{P}^a_{[ij}A^b_{k]}\eta^{ijk}_c + \frac{1}{2}\eta^a\eta^b\mathcal{P}_c\right). \end{aligned} \tag{4.22}$$

Finally, using the equation (4.6) for $J=1$, we generate $\overset{(0)}{\omega}_1$ under the form

$$\overset{(0)}{\omega}_1 = W_{ab}\left(\eta^a H^b_0 - A^{ai}C^b_i\right) + M^c_{ab}\left(A^a_i\eta^b B^{0i}_c - A^a_i A^b_j\eta^{ij}_c\right). \tag{4.23}$$

In consequence, we succeeded in finding the complete form of the first-order deformation of the BRST charge for the model under study, which reduces to the sum among the right hand-sides of the formulas (4.13) and (4.19 –4.23).

4.2 Higher-order deformations

The next target is to investigate the consistency of the first-order deformation of the BRST charge, described by the equation (2.5). By direct computation, it follows that $[\Omega_1,\Omega_1] = \int d^{n-1}x\,\Delta$, with

$$\begin{aligned}\Delta =& K^{abc}t_{abc} + \sum_{k=1}^{n-1}K^{abc}_{a_1\cdots a_k}\frac{\delta^k t_{abc}}{\delta\varphi_{a_1}\cdots\delta\varphi_{a_k}} + \\ &+K^{abc}_{d,}t^d_{abc} + \sum_{k=1}^{n-1}K^{abc}_{d,a_1\cdots a_k}\frac{\delta^k t^d_{abc}}{\delta\varphi_{a_1}\cdots\delta\varphi_{a_k}}, \end{aligned} \tag{4.24}$$

where we made the notations

$$t_{abc} = W_{ec}M^e_{ab} + W_{ea}\frac{\delta W_{bc}}{\delta\varphi_e} + W_{eb}\frac{\delta W_{ca}}{\delta\varphi_e}, \tag{4.25}$$

$$t^d_{abc} = W_{e[a}\frac{\delta M^d_{bc]}}{\delta\varphi_e} + M^d_{e[a}M^e_{bc]}. \tag{4.26}$$

On the one hand, the objects K^{abc}, $K^{abc}_{d,}$, $K^{abc}_{a_1\cdots a_k}$ and $K^{abc}_{d,a_1\cdots a_k}$ are polynomials of ghost number two involving the (undifferentiated) ghosts, antighosts, and fields B^{0i}_a and A^a_i, such that they are not BRST-exact. For instance, the terms corresponding to $k=n-1$ in (4.24) have the concrete form

$$K^{abc}_{a_1\cdots a_{n-1}} = (-)^n P^{i_1}_{a_1}\cdots P^{i_{n-1}}_{a_{n-1}}\eta^a\eta^b C^c_{i_1\cdots i_{n-1}}, \tag{4.27}$$

$$K^{abc}_{d,a_1\cdots a_{n-1}} = \frac{1}{3}P^{i_1}_{a_1}\cdots P^{i_{n-1}}_{a_{n-1}}\eta^a\eta^b\eta^c\eta_{di_1\cdots i_{n-1}}. \tag{4.28}$$

The general form of the functions K^{abc}, $K^{abc}_{d,}$, $K^{abc}_{a_1\cdots a_k}$ and $K^{abc}_{d,a_1\cdots a_k}$ is complicated and not illuminating for subsequent purposes. On the other hand, the equation (2.5) requires that

$[\Omega_1, \Omega_1]$ is s-exact. However, since none of the terms in (4.24) is so, Δ must vanish. This takes place if and only if the following equations are simultaneously obeyed

$$t_{abc} = 0\,, \qquad t^{d}_{abc} = 0\,. \tag{4.29}$$

Analysing the structure of (4.25)–(4.26), we reach the conclusion that the solution to (4.29) reads as

$$M^{c}_{ab} = \frac{\delta W_{ab}}{\delta \varphi_c}\,, \tag{4.30}$$

where, in addition, the now antisymmetric 'two-tensor' W_{ab} is imposed to fulfill the identity

$$W_{e[a}\frac{\delta W_{bc]}}{\delta \varphi_e} = 0 \Leftrightarrow W_{ea}\frac{\delta W_{bc}}{\delta \varphi_e} + \text{cyclic}\,(a, b, c) = 0\,. \tag{4.31}$$

Under these conditions, we can further take $\Omega_2 = 0$, the remaining higher-order deformation equations being satisfied with the choice

$$\Omega_k = 0\,, \qquad k > 2\,. \tag{4.32}$$

In consequence, the consistency of the first-order deformation of the BRST charge for the free model under discussion implements two types of conditions. First, it restricts that W_{ab} and M^{c}_{ab} are no longer independent, but related like in (4.30). This immediately forces the antisymmetry of $W_{ab}\,(\varphi_a)$ with respect to its collection indices since M^{c}_{ab} was already antisymmetric. Second, the antisymmetric 'two-tensor' is constrained to verify the identity (4.31). (We will comment more on the interpretation of these results at the end of section 6.) In this way the complete deformation of the BRST charge, consistent to all orders in the coupling constant, reduces to

$$\Omega = \Omega_0 + g\Omega_1 = \Omega_0 + g \int d^{n-1}x \sum_{k=0}^{n-1} \overset{(k)}{\omega}_1\,, \tag{4.33}$$

where $\overset{(k)}{\omega}_1$ are read from (4.13) and (4.19 –4.23), with $M^{c}_{ab} = \delta W_{ab}/\delta \varphi_c$.

5. Deformation of the BRST-invariant hamiltonian

We now turn our attention to the BRST-invariant hamiltonian (3.30), whose deformation is stipulated by the equations (2.9)–(2.10), etc. Like in the previous section, we investigate only local deformations. Initially, we approach the equation (2.9) associated with its first-order deformation. Inserting (3.31) in (2.9) and using (2.4), on behalf of Jacobi's identity for the Dirac bracket, we find the equation $[H_1 - [K_0, \Omega_1]\,, \Omega_0] = 0$, showing that $H_1 - [K_0, \Omega_1]$ is a BRST observable of the free theory. As mentioned in section 3, all BRST observables are also BRST-exact, or, in other words, they belong to the same equivalence class as the trivial observable zero. In consequence, we can take

$$H_1 = [K_0, \Omega_1]\,, \tag{5.1}$$

where the function K_0 is displayed in (3.32). The expression (5.1) offers us the first-order deformation of the BRST-invariant hamiltonian like

$$\begin{aligned} h_1 = & -W_{ab} H^a_\mu A^{b\mu} - \frac{1}{2} M^c_{ab} A^a_\mu A^b_\nu B^{\mu\nu}_c - \\ & - M^c_{ab} \left(\frac{1}{2} B^{ij}_c \eta^a \mathcal{P}^b_{ij} + A^a_0 \mathcal{P}^b_{ij} \eta^{ij}_c + A^a_0 \eta^b \mathcal{P}_c \right) + \\ & + \frac{\delta W_{ab}}{\delta \varphi_c} P^i_c \left(H^a_i \eta^b + C^a_i A^b_0 \right) + \\ & + \frac{\delta M^c_{ab}}{\delta \varphi_d} P_{di} \left(\eta^a A^b_j B^{ij}_c - \eta^a A^b_0 B^{0i}_c + 2 A^a_0 A^b_j \eta^{ij}_c \right) + \\ & + \frac{1}{4} \left(\frac{\delta M^c_{ab}}{\delta \varphi_d} P_{dij} + \frac{\delta^2 M^c_{ab}}{\delta \varphi_d \delta \varphi_e} P_{di} P_{ej} \right) \eta^a \eta^b B^{ij}_c + \\ & + \sum_{k=2}^{n-1} A^a_0 \frac{\partial^L \overset{(k)}{\omega}_1}{\partial \eta^a} , \end{aligned} \tag{5.2}$$

where we made the notation $\partial^L / \partial \eta^a$ for the left derivative with respect to η^a. In (5.2) and further, M^c_{ab} takes the form (4.30). Because all the components $\overset{(k)}{\omega}_1$ are known, it follows that h_1 is completely determined.

Regarding the second-order deformation, we observe that the third term in the equation (2.10) vanishes due to the fact that $\Omega_2 = 0$. Making use of (5.1) and employing Jacobi's identity with respect to the Dirac bracket, it is easy to see that the second term in (2.10) turns into

$$[H_1, \Omega_1] = \frac{1}{2} [K_0, [\Omega_1, \Omega_1]] , \tag{5.3}$$

which vanishes due to the result established in the previous section, according to which $[\Omega_1, \Omega_1] = 0$. Then, we can set

$$H_2 = 0 , \tag{5.4}$$

which attracts that the remaining equations are satisfied for

$$H_k = 0 , \qquad k > 2 . \tag{5.5}$$

As a consequence, we can write the fully deformed BRST-invariant hamiltonian like

$$H_{\mathrm{B}} = H_{0\mathrm{B}} + g H_1 , \tag{5.6}$$

but also, taking into account (3.31), (4.33) and (5.1)

$$H_{\mathrm{B}} = [K_0, \Omega] . \tag{5.7}$$

The last formula confirms the topological behaviour of the interacting model. It stresses that H_{B} is not only invariant with respect to the deformed hamiltonian BRST symmetry, but also exact. This ends the deformation of the BRST-invariant hamiltonian for the free theory under study.

6. Identification of the interacting theory

With the deformed BRST charge and BRST-invariant hamiltonian at hand, in the sequel we identify the hamiltonian formulation of the interacting first-class theory. Putting together the results from the previous two sections, it follows that the complete expression of the deformed BRST charge consistent to all orders in the deformation parameter is

$$
\begin{aligned}
\Omega = \int d^{n-1}x \Bigg(& \eta^{(1)a}\pi^0_a + 2\eta^{(1)ij}_a \pi^a_{ij} - C^{(1)a}_i p^i_a + \\
& + C^a_i \left(\partial^i \varphi_a + g W_{ab} A^{bi}\right) + \\
& + \eta^a \left(-\partial_i B^{0i}_a + g W_{ab} H^b_0 - g\frac{\delta W_{ab}}{\delta \varphi_c} A^b_i B^{0i}_c\right) + \\
& + \eta^{ij}_a \left(-\partial_{[i} A^a_{j]} - g\frac{\delta W_{bc}}{\delta \varphi_a} A^b_i A^c_j\right) + \\
& + C^a_{ij} \left(-\partial^{[i} P^{j]}_a + g\frac{\delta W_{ab}}{\delta \varphi_c} P^{[i}_c A^{bj]} - g W_{ab} \mathcal{P}^{bij}\right) + \\
& + \eta^{ijk}_a \left(\partial_{[i} \mathcal{P}^a_{jk]} - g\frac{\delta W_{bc}}{\delta \varphi_a} \mathcal{P}^b_{[ij} A^c_{k]} + g\frac{\delta^2 W_{bc}}{\delta \varphi_a \delta \varphi_d} P_{d[i} A^b_j A^c_{k]}\right) - \\
& - g\frac{\delta W_{ab}}{\delta \varphi_c} P^i_c \eta^a C^b_i - g\frac{\delta W_{ab}}{\delta \varphi_c}\left(\frac{1}{2}\eta^a \eta^b \mathcal{P}_c + \mathcal{P}^a_{ij}\eta^b \eta^{ij}_c\right) + \\
& + g\frac{\delta^2 W_{bc}}{\delta \varphi_a \delta \varphi_d} P_{di}\left(\frac{1}{2}\eta^b \eta^c B^{0i}_a + 2A^b_j \eta^c \eta^{ij}_a\right) + \\
& + C^a_{ijk}\Bigg(\partial^{[i} P^{jk]}_a + g\frac{\delta W_{ab}}{\delta \varphi_c} P^{[ij}_c A^{k]}_b + g W_{ab} \mathcal{P}^{bijk} + \\
& \qquad + g\frac{\delta W_{ab}}{\delta \varphi_c} \mathcal{P}^{b[ij} P^{k]}_c + g\frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} P^{[i}_c P^j_d A^{bk]}\Bigg) + \\
& + \eta^{ijkl}_a \Bigg(-\partial_{[i} \mathcal{P}^a_{jkl]} + g\frac{\delta W_{bc}}{\delta \varphi_a} A^c_{[i} \mathcal{P}^b_{jkl]} + g\frac{\delta W_{bc}}{\delta \varphi_a} \mathcal{P}^b_{[ij} \mathcal{P}^c_{kl]} + \\
& \qquad + 6g\frac{\delta^2 W_{bc}}{\delta \varphi_a \delta \varphi_d} P_{d[i} \mathcal{P}^b_{jk} A^c_{l]} - g\frac{\delta^2 W_{bc}}{\delta \varphi_a \delta \varphi_d} P_{d[ij} A^b_k A^c_{l]} - \\
& \qquad - g\frac{\delta^3 W_{bc}}{\delta \varphi_a \delta \varphi_d \delta \varphi_e} P_{d[i} P_{ej} A^b_k A^c_{l]}\Bigg) - \\
& - g\left(\frac{\delta W_{ab}}{\delta \varphi_c} P^{ij}_c + \frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} P^i_c P^j_d\right)\eta^a C^b_{ij} - \\
& - g\left(\frac{\delta W_{ab}}{\delta \varphi_c} \mathcal{P}^a_{ijk} - \frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} P_{d[i} \mathcal{P}^a_{ij]}\right)\eta^b \eta^{ijk}_c + \\
& + g\left(\frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} P_{dij} + \frac{\delta^3 W_{ab}}{\delta \varphi_c \delta \varphi_d \delta \varphi_e} P_{di} P_{ej}\right)\left(3\eta^a A^b_k \eta^{ijk}_c + \frac{1}{2}\eta^a \eta^b \eta^{ij}_c\right) + \\
& + \sum_{k=5}^{n-1} (-)^{k-1} \eta^{i_1 i_2 \cdots i_k}_a \partial_{[i_1} \mathcal{P}^a_{i_2 \cdots i_k]} + \\
& + \sum_{k=4}^{n-1} (-)^{k-1} C^a_{i_1 i_2 \cdots i_k} \partial^{[i_1} P^{i_2 \cdots i_k]}_a + g\sum_{k=3}^{n-1} \overset{(k)}{\omega}_1 \Bigg),
\end{aligned}
\qquad (6.1)
$$

while the full BRST-invariant hamiltonian can be written in the form

$$
\begin{aligned}
H_{\mathrm{B}} = \int d^{n-1}x \Bigg(& -H_i^a \left(\partial^i \varphi_a + gW_{ab} A^{bi} \right) + \\
& + \frac{1}{2} B_a^{ij} \left(-\partial_{[i} A_{j]}^a - g \frac{\delta W_{bc}}{\delta \varphi_a} A_i^b A_j^c \right) + \\
& + A_0^a \left(-\partial_i B_a^{0i} + gW_{ab} H_0^b - g \frac{\delta W_{ab}}{\delta \varphi_c} A_i^b B_c^{0i} \right) + \\
& + \eta^{(1)a} \mathcal{P}_a + \eta_a^{(1)ij} \mathcal{P}_{ij}^a + C_i^{(1)a} P_a^i + g \frac{\delta W_{ab}}{\delta \varphi_c} P_c^i \left(H_i^a \eta^b + C_i^a A_0^b \right) - \\
& - g \frac{\delta W_{ab}}{\delta \varphi_c} \left(\frac{1}{2} B_c^{ij} \eta^a \mathcal{P}_{ij}^b + A_0^a \mathcal{P}_{ij}^b \eta_c^{ij} + A_0^a \eta^b \mathcal{P}_c \right) + \\
& + g \frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} P_{di} \left(B_c^{ij} A_j^b \eta^a - \eta^a A_0^b B_c^{0i} + 2 A_0^a A_j^b \eta_c^{ij} \right) + \\
& + \frac{g}{4} \left(\frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} P_d^{ij} + \frac{\delta^3 W_{ab}}{\delta \varphi_c \delta \varphi_d \delta \varphi_e} P_d^i P_e^j \right) \eta^a \eta^b B_{cij} + \\
& + g \sum_{k=2}^{n-1} A_0^a \frac{\partial^L \overset{(k)}{\omega}_1}{\partial \eta^a} \Bigg) .
\end{aligned} \tag{6.2}
$$

By virtue of the discussion from section 3 on the significance of the various terms in the BRST charge and BRST-invariant hamiltonian, at this stage we extract the general features of the coupled model. Thus, the terms of antighost number zero in (6.1) indicate that only the secondary constraints are deformed as

$$\bar{G}_a^{(2)} \equiv -\left(D_i\right)_a{}^b B_b^{0i} + gW_{ab} H_0^b \approx 0 \,, \tag{6.3}$$

$$\bar{G}_{ij}^{(2)a} \equiv -\bar{F}_{ij}^a \approx 0 \,, \tag{6.4}$$

$$\bar{\gamma}_a^{(2)i} \equiv D^i \varphi_a \approx 0 \,, \tag{6.5}$$

where we employed the notations

$$\left(D_i\right)_a{}^b = \delta_a^b \partial_i + g \frac{\delta W_{ac}}{\delta \varphi_b} A_i^c \,, \tag{6.6}$$

$$\bar{F}_{ij}^a = \partial_{[i} A_{j]}^a + g \frac{\delta W_{bc}}{\delta \varphi_a} A_i^b A_j^c \,, \tag{6.7}$$

$$D^i \varphi_a = \partial^i \varphi_a + gW_{ab} A^{bi} \,. \tag{6.8}$$

It is known that the first-class constraints generate gauge transformations. In consequence, the gauge transformations of the interacting theory will change with respect to the initial ones. From the pieces linear in the antighost number one antighosts, as well as quadratic in the pure ghost number one ghosts, we withdraw that some of the Dirac brackets among the new constraint functions are modified as

$$\left[\bar{G}_a^{(2)}, \bar{G}_b^{(2)} \right] = -g \left(\frac{\delta W_{ab}}{\delta \varphi_c} \bar{G}_c^{(2)} - \frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} B_{d0i} \bar{\gamma}_c^{(2)i} \right) , \tag{6.9}$$

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$$\left[\bar{G}_a^{(2)},\bar{G}_{ij}^{(2)b}\right] = g\left(\frac{\delta W_{ac}}{\delta\varphi_b}\bar{G}_{ij}^{(2)c} - \frac{\delta^2 W_{ac}}{\delta\varphi_b\delta\varphi_d}\bar{\gamma}_{d[i}^{(2)}A_{j]}^c\right), \tag{6.10}$$

$$\left[\bar{G}_a^{(2)},\bar{\gamma}_b^{(2)i}\right] = -g\frac{\delta W_{ab}}{\delta\varphi_c}\bar{\gamma}_c^{(2)i}, \tag{6.11}$$

so the gauge algebra of the first-class constraints is non-abelian and, moreover, open. From the elements simultaneously linear in the pure ghost number two ghosts and in the antighost number one antighosts we determine the first-stage reducibility relations

$$\left(\bar{Z}_{i_1i_2i_3}^a\right)_b^{ij}\bar{G}_{ij}^{(2)b} + \left(\bar{Z}_{i_1i_2i_3}^a\right)_i^b\bar{\gamma}_b^{(2)i} = 0, \tag{6.12}$$

$$\left(\bar{Z}_a^{i_1i_2}\right)_b^{ij}\bar{G}_{ij}^{(2)b} + \left(\bar{Z}_a^{i_1i_2}\right)_i^b\bar{\gamma}_b^{(2)i} = 0, \tag{6.13}$$

where the accompanying reducibility functions read as

$$\left(\bar{Z}_{i_1i_2i_3}^a\right)_b^{ij} = \frac{1}{2}\left(D_{[i_1}\right)^a{}_b\,\delta_{i_2}^i\delta_{i_3]}^j, \tag{6.14}$$

$$\left(\bar{Z}_{i_1i_2i_3}^a\right)_i^b = g\frac{\delta^2 W_{cd}}{\delta\varphi_a\delta\varphi_b}g_{i[i_1}A_{i_2}^cA_{i_3]}^d, \tag{6.15}$$

$$\left(\bar{Z}_a^{i_1i_2}\right)_b^{ij} = -\frac{1}{2}gW_{ab}\left(g^{i_1i}g^{i_2j} - g^{i_1j}g^{i_2i}\right), \tag{6.16}$$

$$\left(\bar{Z}_a^{i_1i_2}\right)_i^b = -\left(D^{[i_1}\right)_a{}^b\,\delta_i^{i_2]}, \tag{6.17}$$

with

$$\left(D_i\right)^a{}_b = \delta_b^a\partial_i - g\frac{\delta W_{bc}}{\delta\varphi_a}A_i^c. \tag{6.18}$$

The part linear in the ghosts with pure ghost number $k+1\geq 3$ contains polynomials of antighost number $k\geq 2$ more than linear in the antighosts, which shows that the reducibility relations of order $k\geq 2$ hold on-shell. Indeed, from the inspection of this type of expressions, we find at pure ghost number three ($k+1=3$) the on-shell second-stage reducibility relations

$$\begin{aligned}\left(\bar{Z}_{i_1i_2i_3i_4}^a\right)_b^{j_1j_2j_3}\left(\bar{Z}_{j_1j_2j_3}^b\right)_c^{ij}f_{ij}^c + \left(\bar{Z}_{i_1i_2i_3i_4}^a\right)_{j_1j_2}^b\left(\bar{Z}_b^{j_1j_2}\right)_c^{ij}f_{ij}^c &= -g\left(\frac{\delta W_{bc}}{\delta\varphi_a}\bar{G}_{[i_1i_2}^{(2)b}f_{i_3i_4]}^c -\right.\\ &\left.- \frac{\delta^2 W_{cd}}{\delta\varphi_a\delta\varphi_b}\bar{\gamma}_{b[i_1}^{(2)}A_{i_2}^cf_{i_3i_4]}^d\right),\end{aligned} \tag{6.19}$$

$$\begin{aligned}\left(\bar{Z}_a^{i_1i_2i_3}\right)_{j_1j_2}^b\left(\bar{Z}_b^{j_1j_2}\right)_i^c f_c^i + \left(\bar{Z}_a^{i_1i_2i_3}\right)_b^{j_1j_2j_3}\left(\bar{Z}_{j_1j_2j_3}^b\right)_i^c f_c^i &= g\left(\frac{\delta W_{ab}}{\delta\varphi_c}\bar{G}^{(2)b[i_1i_2}f_c^{i_3]} -\right.\\ &\left.- \frac{\delta^2 W_{ab}}{\delta\varphi_c\delta\varphi_d}\bar{\gamma}_c^{(2)[i_1}A^{bi_2}f_d^{i_3]}\right),\end{aligned} \tag{6.20}$$

where f_c^i and f_{ij}^c are arbitrary smooth functions (the latter are antisymmetric in their spatial indices), along with the second-stage reducibility functions

$$\left(\bar{Z}_{i_1i_2i_3i_4}^a\right)_b^{j_1j_2j_3} = -\frac{1}{3!}\left(D_{[i_1}\right)^a{}_b\,\delta_{i_2}^{j_1}\delta_{i_3}^{j_2}\delta_{i_4]}^{j_3}, \tag{6.21}$$

$$\left(\bar{Z}_{i_1i_2i_3i_4}^a\right)_{j_1j_2}^b = -\frac{g}{2}g_{j_1k_1}g_{j_2k_2}\frac{\delta^2 W_{cd}}{\delta\varphi_a\delta\varphi_b}\delta_{[i_1}^{k_1}\delta_{i_2}^{k_2}A_{i_3}^cA_{i_4]}^d, \tag{6.22}$$

$$\left(\bar{Z}_a^{i_1i_2i_3}\right)^b_{j_1j_2} = \frac{1}{2}\left(D^{[i_1}\right)_a^{\ b}\delta^{i_2}_{j_1}\delta^{i_3]}_{j_2}\,, \tag{6.23}$$

$$\left(\bar{Z}_a^{i_1i_2i_3}\right)^{j_1j_2j_3}_b = \frac{g}{3!}W_{ab}\sum_{\sigma\in S_3}(-)^\sigma g^{i_1j_{\sigma(1)}}g^{i_2j_{\sigma(2)}}g^{i_3j_{\sigma(3)}}\,. \tag{6.24}$$

In (6.24) S_3 signifies the set of permutations of $\{1,2,3\}$, and $(-)^\sigma$ means the parity of a certain permutation σ pertaining to S_3. By making a similar analysis with respect to the terms linear in the pure ghost number $(p+1)$ ghosts $(p=3,\ldots,n-3)$, we extract the on-shell p-stage reducibility relations

$$\left(\bar{Z}^a_{i_1\cdots i_{p+2}}\right)^{j_1\cdots j_{p+1}}_b\left(\bar{Z}^b_{j_1\cdots j_{p+1}}\right)^{k_1\cdots k_p}_c+\left(\bar{Z}^a_{i_1\cdots i_{p+2}}\right)^b_{j_1\cdots j_p}\left(\bar{Z}^{j_1\cdots j_p}_b\right)^{k_1\cdots k_p}_c\approx 0\,, \tag{6.25}$$

$$\left(\bar{Z}_a^{i_1\cdots i_{p+1}}\right)^b_{j_1\cdots j_p}\left(\bar{Z}_b^{j_1\cdots j_p}\right)^c_{k_1\cdots k_{p-1}}+\left(\bar{Z}_a^{i_1\cdots i_{p+1}}\right)^{j_1\cdots j_{p+1}}_b\left(\bar{Z}^b_{j_1\cdots j_{p+1}}\right)^c_{k_1\cdots k_{p-1}}\approx 0\,, \tag{6.26}$$

plus the p-stage reducibility functions

$$\left(\bar{Z}^a_{i_1\cdots i_{p+2}}\right)^{j_1\cdots j_{p+1}}_b = \frac{(-)^{p+1}}{(p+1)!}\left(D_{[i_1}\right)^a_{\ b}\delta^{j_1}_{i_2}\cdots\delta^{j_{p+1}}_{i_{p+2}]}\,, \tag{6.27}$$

$$\left(\bar{Z}^a_{i_1\cdots i_{p+2}}\right)^b_{j_1\cdots j_p} = -\frac{(-)^p g}{p!}g_{j_1k_1}\cdots g_{j_pk_p}\frac{\delta^2W_{cd}}{\delta\varphi_a\delta\varphi_b}\delta^{k_1}_{[i_1}\cdots\delta^{k_p}_{i_p}A^c_{i_{p+1}}A^d_{i_{p+2}]}\,, \tag{6.28}$$

$$\left(\bar{Z}_a^{i_1\cdots i_{p+1}}\right)^b_{j_1\cdots j_p} = \frac{(-)^p}{p!}\left(D^{[i_1}\right)_a^{\ b}\delta^{i_2}_{j_1}\cdots\delta^{i_{p+1}]}_{j_p}\,, \tag{6.29}$$

$$\left(\bar{Z}_a^{i_1\cdots i_{p+1}}\right)^{j_1\cdots j_{p+1}}_b = \frac{(-)^p g}{(p+1)!}W_{ab}\sum_{\sigma\in S_{p+1}}(-)^\sigma g^{i_1j_{\sigma(1)}}g^{i_2j_{\sigma(2)}}\cdots g^{i_{p+1}j_{\sigma(p+1)}}\,. \tag{6.30}$$

In (6.30) S_{p+1} and $(-)^\sigma$ denote the set of permutations of $\{1,2,\ldots,p+1\}$, respectively, the parity of a permutation σ pertaining to S_{p+1}. Finally, the elements linear in the pure ghost number $(n-1)$ ghosts describe the reducibility relations of highest order

$$\begin{aligned}&\left(\bar{Z}_a^{i_1\cdots i_{n-1}}\right)^b_{j_1\cdots j_{n-2}}\left(\bar{Z}_b^{j_1\cdots j_{n-2}}\right)^c_{k_1\cdots k_{n-3}}f_c^{k_1\cdots k_{n-3}}+\\&+\left(\bar{Z}_a^{i_1\cdots i_{n-1}}\right)^{j_1\cdots j_{n-1}}_b\left(\bar{Z}^b_{j_1\cdots j_{n-1}}\right)^c_{k_1\cdots k_{n-3}}f_c^{k_1\cdots k_{n-3}} = -g\left(\frac{\delta^2W_{ab}}{\delta\varphi_c\delta\varphi_d}\bar{\gamma}_c^{(2)[i_1}A^{bi_2}f_d^{i_3\cdots i_{n-1}]}-\right.\\&\qquad\qquad\left.\frac{\delta W_{ab}}{\delta\varphi_c}\bar{G}^{(2)b[i_1i_2}f_c^{i_3\cdots i_{n-1}]}\right),\end{aligned} \tag{6.31}$$

$$\begin{aligned}&\left(\bar{Z}_a^{i_1\cdots i_{n-1}}\right)^b_{j_1\cdots j_{n-2}}\left(\bar{Z}_b^{j_1\cdots j_{n-2}}\right)^{k_1\cdots k_{n-2}}_c f^c_{k_1\cdots k_{n-2}}\\&+\left(\bar{Z}_a^{i_1\cdots i_{n-1}}\right)^{j_1\cdots j_{n-1}}_b\left(\bar{Z}^b_{j_1\cdots j_{n-1}}\right)^{k_1\cdots k_{n-2}}_c f^c_{k_1\cdots k_{n-2}} = -g\frac{\delta W_{ab}}{\delta\varphi_c}\bar{\gamma}_c^{(2)[i_1}f^{bi_2\cdots i_{n-1}]}\,,\end{aligned} \tag{6.32}$$

where $f_c^{k_1\cdots k_{n-3}}$ and $f^c_{k_1\cdots k_{n-2}}$ are arbitrary completely antisymmetric smooth functions, and equally furnish the $(n-2)$-order reducibility functions

$$\left(\bar{Z}_a^{i_1\cdots i_{n-1}}\right)^b_{j_1\cdots j_{n-2}} = \frac{(-)^n}{(n-2)!}\left(D^{[i_1}\right)_a^{\ b}\delta^{i_2}_{j_1}\cdots\delta^{i_{n-1}]}_{j_{n-2}}\,, \tag{6.33}$$

$$\left(\bar{Z}_a^{i_1\cdots i_{n-1}}\right)^{j_1\cdots j_{n-1}}_b = \frac{(-)^n g}{(n-1)!}W_{ab}\sum_{\sigma\in S_{n-1}}(-)^\sigma g^{i_1j_{\sigma(1)}}g^{i_2j_{\sigma(2)}}\cdots g^{i_{n-1}j_{\sigma(n-1)}}\,. \tag{6.34}$$

The notations S_{n-1} and $(-)^{\sigma}$ are similar with the above ones. This is of course not all the information we gain on the interacting first-class theory. We actually know everything on the tensor structure of the deformed first-class constraints from (6.1) if we merely separate specific polynomials in the ghosts and antighosts. For example, the relations (6.9)–(6.11) underline that the gauge algebra of the deformed first-class constraints is open, and, meanwhile, display the concrete form of the first-order structure functions. However, there is a tower of higher-order structure functions, that satisfy recursive equations, dictated in the hamiltonian formulation by taking their repeated Dirac brackets with the first-class constraint functions. These equations will have an intricate form due to the fact that the interacting model is also on-shell reducible. From (6.1) we can precisely withdraw these higher-order structure functions, as well as the equations that relate them at any level, if we just isolate the appropriate polynomials in the ghosts and antighosts.

Now, we investigate the modified BRST-invariant hamiltonian (6.2). The component of antighost number zero

$$H = \int d^{n-1}x \left(-H_i^a \bar{\gamma}_a^{(2)i} + \frac{1}{2} B_a^{ij} \bar{G}_{ij}^{(2)a} + A_0^a \bar{G}_a^{(2)} \right), \tag{6.35}$$

represents nothing but the new first-class hamiltonian, while the terms linear in the antighost number one antighosts give the deformed gauge algebra relations

$$\left[H, G_a^{(1)} \right] = \bar{G}_a^{(2)}, \tag{6.36}$$

$$\begin{aligned} \left[H, \bar{G}_a^{(2)} \right] &= g \frac{\delta W_{ab}}{\delta \varphi_c} \left(A_0^b \bar{G}_c^{(2)} - H_i^b \bar{\gamma}_c^{(2)i} - \frac{1}{2} \bar{G}_{ij}^{(2)b} B_c^{ij} \right) + \\ &\quad + g \frac{\delta^2 W_{ac}}{\delta \varphi_b \delta \varphi_d} \left(\frac{1}{2} B_b^{ij} \bar{\gamma}_{d[i}^{(2)} A_{j]}^c - B_{d0i} A_0^c \bar{\gamma}_b^{(2)i} \right), \end{aligned} \tag{6.37}$$

$$\left[H, G_{ij}^{(1)a} \right] = \bar{G}_{ij}^{(2)a}, \tag{6.38}$$

$$\left[H, \bar{G}_{ij}^{(2)a} \right] = g \left(\frac{\delta W_{bc}}{\delta \varphi_a} A_0^b \bar{G}_{ij}^{(2)c} - \frac{\delta^2 W_{cd}}{\delta \varphi_a \delta \varphi_b} A_0^c \bar{\gamma}_{b[i}^{(2)} A_{j]}^d \right), \tag{6.39}$$

$$\left[H, \gamma_a^{(1)i} \right] = \bar{\gamma}_a^{(2)i}, \tag{6.40}$$

$$\left[H, \bar{\gamma}_a^{(2)i} \right] = g \frac{\delta W_{ab}}{\delta \varphi_c} A_0^b \bar{\gamma}_c^{(2)i}. \tag{6.41}$$

Just like in the case of the BRST charge, the formula (6.2) tells us everything on the tensor structure of the interacting hamiltonian (6.35) that governs the dynamics on the deformed first-class surface. Indeed, from (6.36)–(6.41) we learn, besides the first-class behaviour of H, that there appear some nontrivial structure functions. This means that there will also be a consequent recursive 'open setting', formulated in terms of higher-order structure functions of H and of the equations that relate them, that can be derived by taking the repeated Dirac brackets of (6.36)–(6.41) with the deformed first-class constraint functions. Of course, the on-shell reducibility of the new first-class constraints will be involved at every stage. We have this entire setting at our hand, and can write it down at any level, simply by identifying the adequate polynomials in the ghosts and antighosts from (6.2).

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So far, it is clear that the entire hamiltonian deformation is controlled by W_{ab} since if we set $W_{ab}=0$ we recover the initial free topological field theory even when the coupling constant is different from zero. (In other words, we get no deformations at all.) Moreover, we have seen that the consistency of the deformation restricts $W_{ab}(\varphi)$ to be antisymmetric and to satisfy the identity (4.31). Let us see the geometric meaning of this so-called 'two-tensor'. To this end, we briefly review the basic notions on Poisson manifolds. If N denotes an arbitrary Poisson manifold, then this is equipped with a Poisson bracket $\{,\}$ that is bilinear, antisymmetric, subject to a Leibnitz-like rule and satisfies a Jacobi-type identity. If $\left\{X^i\right\}$ are some local coordinates on N, then there exists a two-tensor $\mathcal{P}^{ij}\equiv\left\{X^i,X^j\right\}$ (the Poisson tensor) that uniquely determines the Poisson structure together with the Leibnitz rule. This two-tensor is antisymmetric and transforms covariantly under coordinate transformations. Jacobi's identity for the Poisson bracket $\{,\}$ expressed in terms of the Poisson tensor reads as $\mathcal{P}^{ij}_{,k}\mathcal{P}^{kl}+\text{cyclic}\,(i,j,l)=0$, where $\mathcal{P}^{ij}_{,k}\equiv\partial\mathcal{P}^{ij}/\partial X^k$. Now, the geometric origin of W_{ab} is obvious. If, for instance, we choose a concrete form for the antisymmetric functions $W_{ab}(\varphi)$ that satisfy (4.31), then we can interpret the dynamical scalar fields $\{\varphi_a\}$ precisely like some local coordinates on a target manifold endowed with a prescribed Poisson structure (up to the plain convention that the lower index a is a 'covariant' index of the type i). Conversely, any given Poisson manifold parametrized in terms of some local coordinates $\{\varphi_a\}$ (within the same index convention) prescribes a Poisson tensor $W_{ab}(\varphi)$ which is antisymmetric and satisfies (4.31). This discussion also argues that the attribute of 'two-tensor' given to W_{ab} is not misleading, but only hidden behind some Poisson structure. It is clear that for an odd number of scalar fields, W_{ab} is degenerate irrespective of its concrete form. For an even number of scalar fields, we can find nondegenerate forms of W_{ab}, whose inverse will be nothing but the symplectic two-form on the target space, which becomes a symplectic manifold.

Passing to the lagrangian formulation of the interacting theory, after some computation we get the action

$$S\left[A^a_\mu,H^a_\mu,\varphi_a,B^{\mu\nu}_a\right]=\int d^nx\left(H^a_\mu D^\mu\varphi_a+\frac{1}{2}B^{\mu\nu}_a\bar{F}^a_{\mu\nu}\right), \tag{6.42}$$

subject to the gauge invariances

$$\delta_\epsilon A^a_\mu = \left(D^\mu\right)^a{}_b\,\epsilon^b\,, \tag{6.43}$$

$$\delta_\epsilon\varphi_a = -gW_{ab}\epsilon^b\,, \tag{6.44}$$

$$\begin{aligned}\delta_\epsilon H^a_\mu &= \left(D^\nu\right)^a{}_b\,\epsilon^b_{\mu\nu}-g\frac{\delta W_{bc}}{\delta\varphi_a}\epsilon^b H^c_\mu+\\ &\quad+g\frac{\delta^2 W_{cd}}{\delta\varphi_a\delta\varphi_b}\left(\frac{1}{2}A^{c\nu}A^{d\rho}\epsilon_{b\mu\nu\rho}+A^{d\nu}\epsilon^c B_{b\mu\nu}\right),\end{aligned} \tag{6.45}$$

$$\delta_\epsilon B^{\mu\nu}_a = \left(D_\rho\right)_a{}^b\,\epsilon^{\mu\nu\rho}_b+gW_{ab}\epsilon^{b\mu\nu}-g\frac{\delta W_{ab}}{\delta\varphi_c}\epsilon^b B^{\mu\nu}_c, \tag{6.46}$$

where $D^\mu\varphi_a$, $\bar{F}^a_{\mu\nu}$, $\left(D^\mu\right)^a{}_b$ and $\left(D_\rho\right)_a{}^b$ can be read from the formulas (6.6)–(6.8) and (6.18) by manifest Lorentz covariance. The deformation of the lagrangian gauge transformations

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roots in the deformed first-class constraints (6.3)–(6.5). It can be shown that these gauge transformations are on-shell $(n-2)$ -order reducible and give rise to an open gauge algebra.

We notice that neither the lagrangian action, nor the gauge symmetry of the interacting theory, do contain the n-dimensional antisymmetric symbol. This is a direct consequence of the fact that we have removed the term containing the spatial part of this symbol from the last component in the first-order deformation of the BRST charge. Indeed, such a term would have resulted at the level of the lagrangian action in the vertex

$$\varepsilon^{\mu_1\cdots\mu_n} U_{a_1\cdots a_n} A^{a_1}_{\mu_1} \cdots A^{a_n}_{\mu_n}\,, \tag{6.47}$$

that breaks the PT-invariance. In (6.47), the functions $U_{a_1\cdots a_n}$ involve only the undifferentiated scalar fields, are completely antisymmetric in their indices, and are required to satisfy some identities implied by the consistency of the first-order deformation of the BRST charge. This type of interactions will be reported elsewhere.

7. Conclusion

In conclusion, in this paper we have generated the consistent hamiltonian interactions in any spacetime dimension $n \geq 4$ that can be introduced among a set of scalar fields, two types of one-forms and a system of two-forms, pictured in the free limit by an abelian topological field theory of BF-type. Our treatment is mainly based on the hamiltonian BRST deformation procedure, that relies on the construction of the consistent deformations of both BRST charge and BRST-invariant hamiltonian of the free model with the help of some cohomological techniques. In addition, we require that the deformations are local and independent of the spacetime dimension. The results regarding the deformation of the BRST charge can be synthesized by the fact that only the first-order deformation can be taken as nonvanishing, while its consistency reveals some functions on the undifferentiated scalar fields that can be seen as the components of a Poisson two-tensor on the target space. Concerning the deformation of the BRST-invariant hamiltonian, it stops at order one in the coupling constant as well, and, moreover, is exact with respect to the deformed hamiltonian BRST symmetry (see (5.7)). From these two deformed quantities we derive the hamiltonian formulation of the resulting coupled model, namely, its first-class constraints, accompanying reducibility functions, first-class hamiltonian and gauge algebra relations. This is an example of deformation that modifies the gauge transformations, the reducibility relations, and also the gauge algebra. The resulting model is included precisely within the class of interacting topological field theories of BF-type with an open hamiltonian gauge algebra and on-shell reducibility relations. This work generalizes our previous results from [7]–[8] in the sense that although the gauge structure of the interacting model is richer, the lagrangian of the interacting theory has a similar expression. We mention that the two-dimensional case studied in [7] is irreducible and, in fact, equivalent to the standard Poisson Sigma Model [15], up to the fact that it is written in more complicated variables, but this equivalence is no longer valid in $n \geq 4$ dimensions, where it can be observed a complex structure of new nontrivial terms.

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